Heat Conduction in Slabs with Uniform Heat Sources and Radiative Boundary Conditions

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Theme

A FINITE-difference technique is employed to determine the transient temperature distribution in a plane slab, with uniform heat sources and subjected to radiative cooling; the solid has a uniform initial temperature, and the heat conduction is one-dimensional. Results are presented in dimensionless charts over a wide range of physical parameters. Under various limiting conditions, the results are found in excellent agreement with the analytical solutions.

Content

The transient heat conduction in a solid with internal heat sources and radiation boundary condition still does not have complete solutions. Problems of this type occur frequently in the components of nuclear devices operating in the atmosphere or space, such as nuclear ramjets and rockets. Studies of one-dimensional, nonlinear heat conduction problems using analog and digital computers were presented by many investigators.[1–4] However, none of these investigations have taken into account the internal heat generation, which can be an important consideration in engineering application. Recently, attempts have been made by Chung and Yeh[5] and Zyszkowski[6] to include the internal heat generations in their approximate nonlinear heat-transfer analyses. However, the former restricted their analysis to a semi-infinite solid, the latter yielded solutions which are not applicable for the case of radiative heating. Solutions of a general nature have not yet been established due to their complexity. The purpose of this work is to employ a finite-difference technique to determine the transient temperature distributions in a plane slab with internal heat sources and nonlinear radiative boundary conditions.

Governing equations. Consideration is given to a plane slab which is initially at a uniform temperature, T_i, and then is suddenly $[\theta = 0+]$ exposed to radiant heat transfer in one surface, $[x' = L]$, while the other side $[x' = 0]$ is insulated. Furthermore, the radiation interchange factor $\mathscr{F}$ is assumed to be independent of surface temperature of the solid; the environment temperature T_e is constant and the internal heat generation g_0 is uniformly distributed. If the conductivity is assumed to be temperature-independent, the relevant nondimensional conduction equation and the associated boundary conditions may be written as

$$\partial U/\partial t = \partial^2 U/\partial X^2 + Q \qquad (1)$$

$$U = U_i \qquad X \geq 0 \quad t = 0 \qquad (2)$$

$$-\partial U/\partial X = (U^4 - U_e^4)/H \qquad X = 1 \quad t > 0 \qquad (3)$$

and

$$\partial U/\partial X = 0 \qquad X = 0 \quad t > 0 \qquad (4)$$

Received November 2, 1972; synoptic received February 2, 1973; revision received March 5, 1973. Full paper available from National Technical Information Service, Springfield, Va., 22151, as N73-19944 at the standard price (available upon request).
Index categories: Heat Conduction; Radiation Heat Transfer.

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Table 1 Significant parameters

Parameters	Symbols	Values
Temperature	$V = T - T_i/T_e - Ti$ ‡	Continuous but varies
Time	$t = \alpha\theta/L^2$	Continuous from 0.001 to 10
Position	$X = x'/L$	0, 0.5, 1.0
Radiation number	$N_{rc} = K/\sigma\mathscr{F}T_i^3L$ ‡	0.1, 1.0, 5.0, 30
Generation	$Q = g_0L^2/KT_i$ ‡	1, 5, 10
Temperature ratio	$U_e = T_e/T_i$ ‡	0, 0.25, 0.5, 0.75

where $U = T/T_i$, $H = K/\sigma\mathscr{F}T_i{}^3L$ for radiant cooling,‡ $U_i = 1$ for cooling, $U_i = T_i/T_e$ for heating, $U_e = 1$ for heating, and $U_e = T_e/T_i$ for cooling.

Temperature history charts. Equations (1–4) are solved numerically using an explicit finite-difference scheme. Results of temperature distribution are presented graphically in dimensionless form. In this study, seven parameters are chosen and their ranges are presented in Table 1.

Figures 1–3 show typical examples of temperature distribution for radiant cooling when T_e/T_i is equal to 0.25. It is noted that a complete different trend of temperature distribution is found for the case of radiant heating.

Effect of radiation number. An inspection of Figs. 1–3 reveals that the time required to reach the "steady state" for cooling a slab increases as the radiation number increases. Similarly, the time required to reach some particular temperature in heating a slab increases as the radiation number increases. At small time, the temperature at the insulated surface is insensitive to the radiation number. When the radiation is less than 10^{-5}, the analytical solution obtained on the basis of step jump surface temperature boundary conditions can be used. For a very large value of radiation number, the temperature inside the solid can be considered uniformly distributed. Present calculations show that at $N_{rc} = 90{,}000$ the numerical results and the corresponding

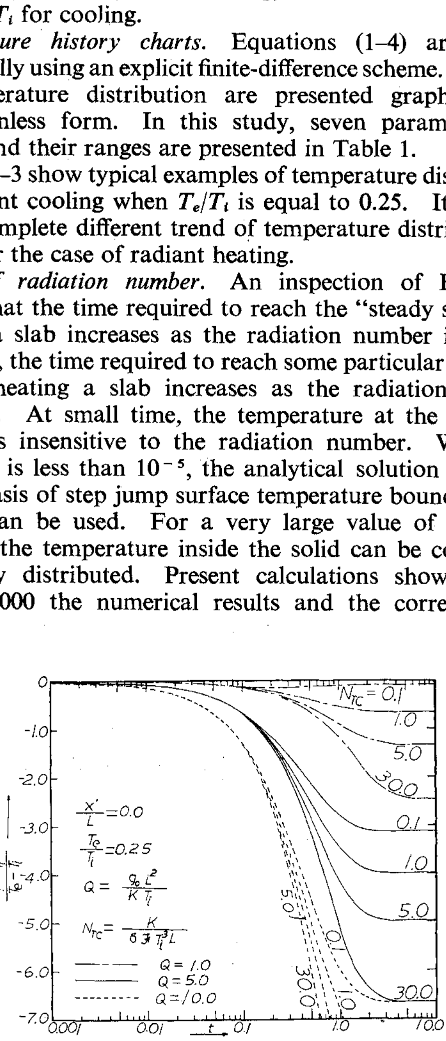

Fig. 1 Temperature history at the insulated surface of a plane slab.

‡ For the case of radiant heating, the corresponding symbol may be obtained by replacing the subscript *e* by *i* and *i* by *e*.

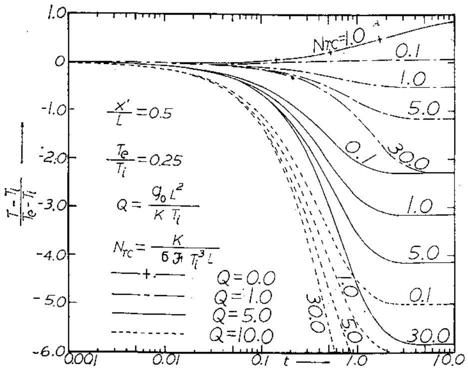

Fig. 2 Temperature history at the central plane of a plane slab.

isothermal solutions coincide if the parameters are $Q = 1$ and $U_e = 0.25$. However, when the heat generation is reduced to 0.1, the isothermal solution becomes valid when the radiation number reaches 5000. This feature differs appreciably from the results of Ref. 1 in which no internal generation is included. In this earlier work, the isothermal solution has been

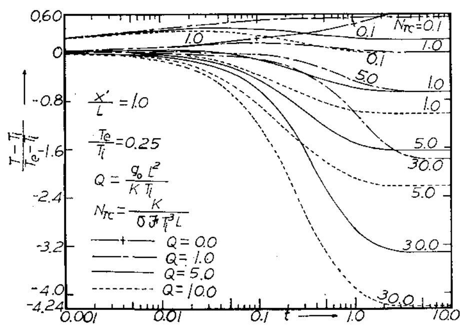

Fig. 3 Temperature history at the radiation surface of a plane slab.

found applicable when the radiation number is greater than 50 at $U_e = 0.25$.

Extreme cases. The present results agree well with those obtained by the previous authors[1-2] when there is no internal heat generation (see Figs. 2 and 3).

For very small values of time, the asymptotic solutions for the radiation surface temperature can be obtained by applying the Laplace transform to Eqs. (1–4) with the aid of convolution theorem. It is found that the numerical solution can be well represented by the asymptotic solution when t is less than 10^{-4}. On the other hand, when t is very large, the temperatures of the slab shown in Figs. 1–3 approach to the corresponding steady solutions.

In the case that the ambient temperature and the initial temperature of the solid are identical, approximate solutions based on a variational method are available.[6] Numerical computations indicate that the maximum deviation of temperature ratio in Ref. 6 from the present work is less than 7%.

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